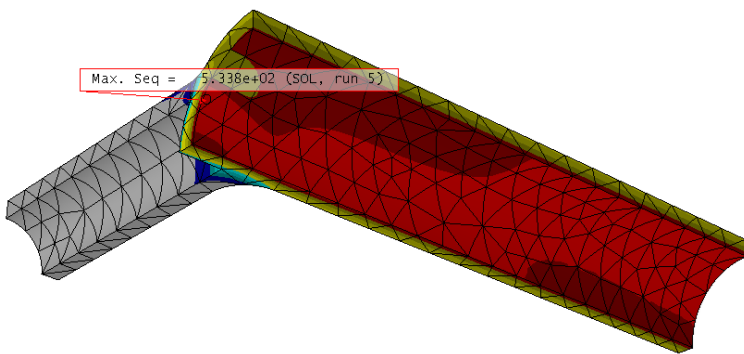
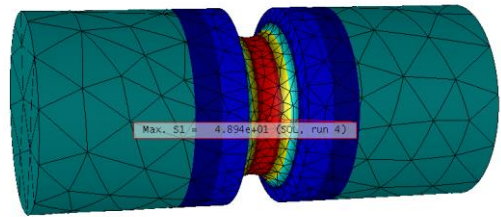


Benchmarking by FEA with StressCheck

January 2020 NAFEMS Linear Benchmarks: StressCheck Solutions



ESRD, Inc.
2020





- ❑ In January 2020, [NAFEMS Benchmark magazine](#) published an article by Technical Officer Ian Symington titled “Designer Oriented Software - Is it Accurate?”
 - The goal of the article was to compare the results of several designer oriented software tools with the target solutions for a set of eight (8) linear benchmarks.
 - About the target solutions (via Symington): “The target solution used for most of the benchmarks in this study has been produced using the traditional Finite Element Analysis (FEA) approach. Confidence in the target solutions is gained using mesh convergence studies. A number of the benchmark examples reference an analytical solution, this information has been included to provide further confidence in the target solution.
 - The download of the article may be found [here](#).
- ❑ StressCheck Professional was then used to solve the NAFEMS benchmarks [via p-extension](#) on automeshed tetrahedra, with the minimum discretization (i.e. mesh density and displacement polynomial orders) required to show convergence of the data of interest (e.g max von Mises stress, Seq, max 1st principal stress, S1, spring rate K, or natural frequencies) **within 1% estimated error.**

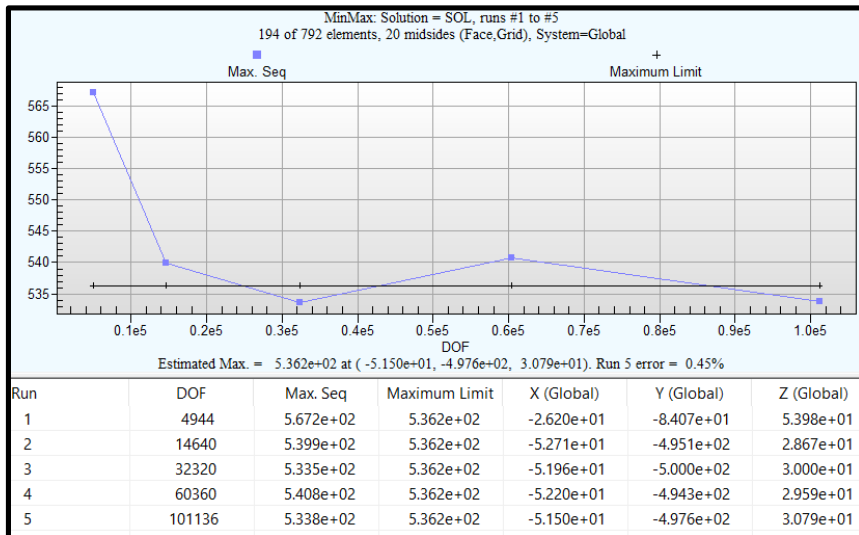
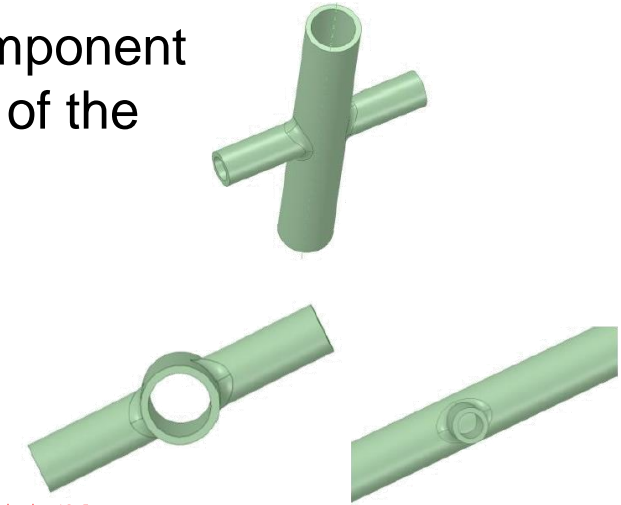


Benchmark 1: Pressure Component

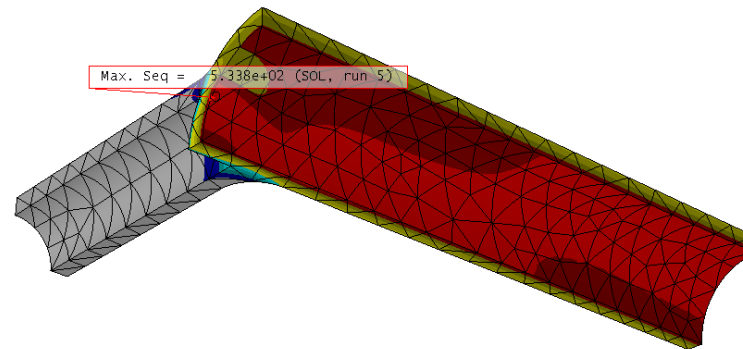


STRESSCHECK

- ❑ Symington: “The pipeline is to be analysed in order to determine if the component is appropriately designed to withstand a 100MPa pressure load. The goal of the benchmark is to predict the peak Von Mises stress in the component.”
- ❑ StressCheck solution (1/8th domain, symmetry boundary conditions):
 - 792 geometrically mapped tetrahedra, p=2 to 6, <1 minute run time
 - Max Seq converges to **534 MPa (with 0.45% estimated error)**



Target max Seq:
534 MPa



StressCheck v10.5
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Min= 2.917e-03

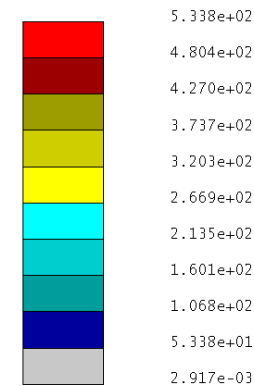


Image courtesy NAFEMS

Benchmark 2: Coil Spring



STRESSCHECK®

- Symington: “Benchmark 2 tests the ability of the package to predict the compliance of a coil spring. The coil spring is a challenging geometry to mesh using a traditional FEA approach. The challenge, target solution and results of the respondents are presented in the NAFEMS Benchmark Article “[How Confident Are You?](#)”
- StressCheck solution (same as the reference for the NAFEMS solution):
 - 9691 quadratically mapped tetrahedra, p=2 to 5, 2 minute run time
 - Spring rate converges to **20.83 N/mm (with 0.05% estimated error)**

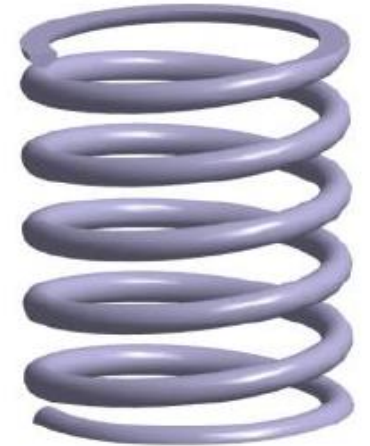


Image courtesy NAFEMS

p	N	U (Nmm)	% error
2	58,67	6558.910	0.78
3	173,989	6515.587	0.12
4	385,200	6512.737	0.07
5	721,473	6511.252	0.05
∞	∞	6507.872	–

Image courtesy NAFEMS/Dr. Barna Szabó

$$U = 6507.87 \text{ N-mm}, \Delta = -25 \text{ mm}$$
$$K = 2 * U / \Delta^2 = 20.83 \text{ N/mm}$$

Target spring rate:
 $K = 20.83 \text{ N/mm}$

Benchmark 3: Skew Plate – Bending Dominated



STRESSCHECK

- Symington: “The third benchmark tackles a thin skewed plate. The skewed plate geometry is designed to introduce complexities in the discretisation process and may introduce distorted elements in an automatic meshing process. The thin skewed plate is simply supported and loaded with a uniform pressure. This benchmark is taken from the [NAFEMS Linear Static Benchmarks Volume 1](#), test number IC 13.”
- StressCheck solution:
 - 64 geometrically mapped quads, $p=1$ to 8, <10 seconds run time
 - Max S1 converges to **0.820 MPa (with 0.05% estimated error) at point E on lower surface**

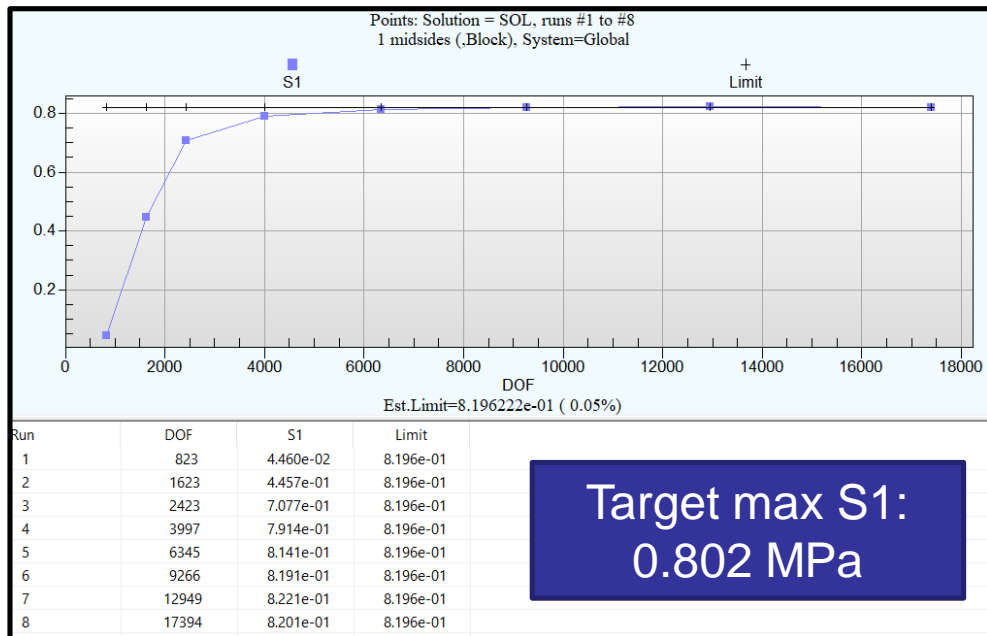
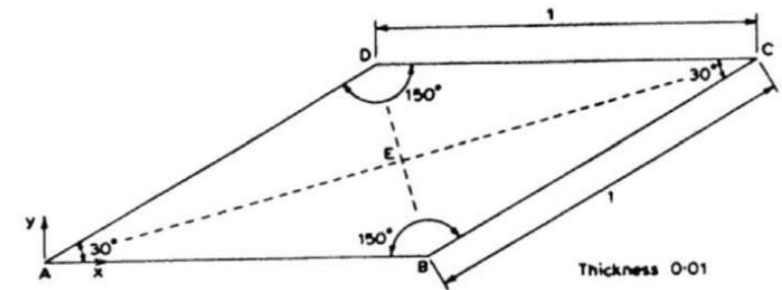


Image courtesy NAFEMS



More details on the StressCheck solution can be found here: <https://www.esrd.com/product/standard-nafems-benchmarks-linear-elastic-tests/>

Benchmark 4: Plate with Hole - Kt



STRESSCHECK

- Symington: “This benchmark tests the ability of the code to capture a stress concentration (Kt) in a plate containing a small hole. The benchmark has been designed so that the extent of the plate is large in comparison to the size of hole so as to pose a challenge when sizing the mesh in the vicinity of the stress concentration.”
- StressCheck solution:
 - 355 geometrically mapped tetrahedra, $p=2$ to 8, 1 minute run time
 - Max S1 converges to **316 Mpa (with 0.00% estimated error)**

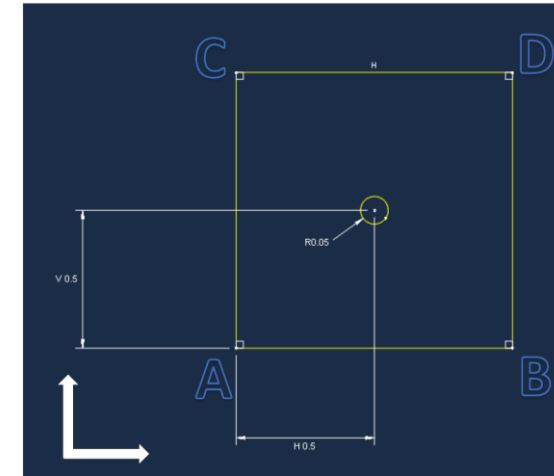
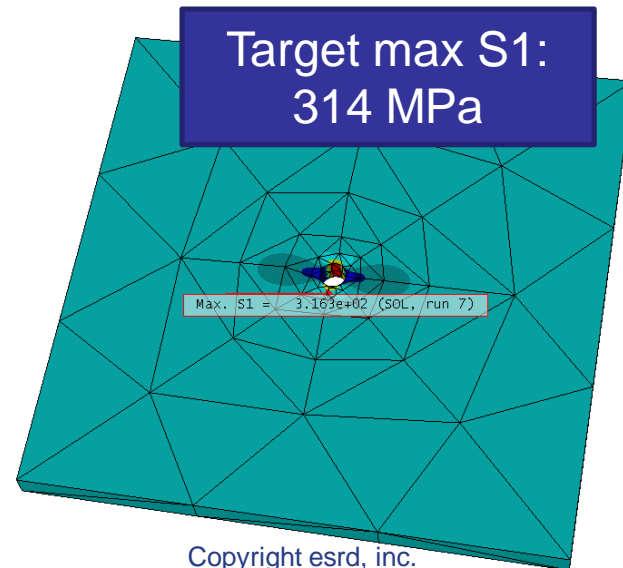
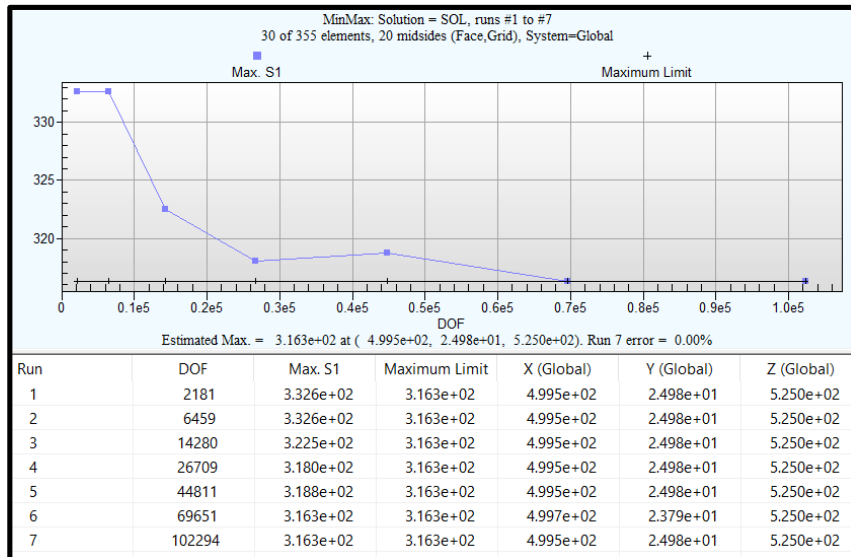


Image courtesy NAFEMS

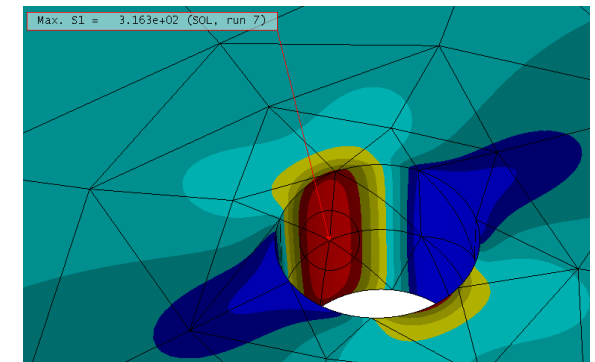


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StressCheck v10.5
Units = MM/N/SEC/C
LINEAR ID=SOL
Run=7, DOF=102294
Fnc.=S1
Max= 3.163e+02
Min= -2.236e+00



3.163e+02
2.844e+02
2.526e+02
2.207e+02
1.889e+02
1.570e+02
1.252e+02
9.332e+01
6.147e+01
2.962e+01
-2.236e+00



Benchmark 5: Shaft with U-Shaped Notch- Kt



STRESSCHECK

- ❑ Symington: “Benchmark 5 is again intended to test if the software package can appropriately capture a stress concentration (Kt). This benchmark uses a circular shaft loaded in uniaxial tension with a U-shaped notch running around the entire circumference of the shaft at midspan.”
- ❑ StressCheck solution:
 - 3833 geometrically mapped tetrahedra, p=2 to 5, 1.5 minute run time
 - Max S1 converges to **48.94 MPa (with 0.24% estimated error)**

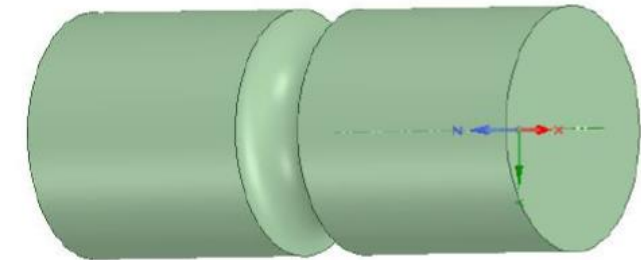
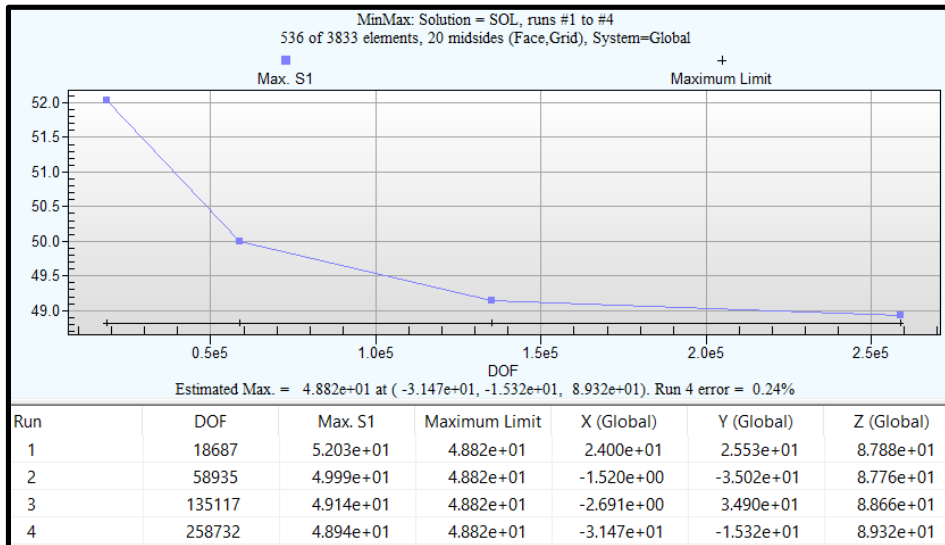
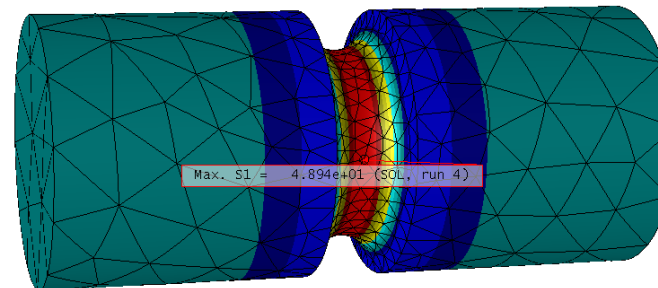


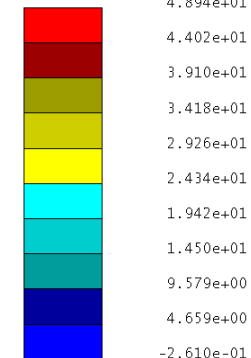
Image courtesy NAFEMS



Target max S1:
48.2 MPa



StressCheck v10.5
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Run=4, DOF=258732
Fnc.=S1
Max= 4.894e+01
Min= -2.610e-01



Benchmark 6: Modal – Cantilevered Thin Plate



STRESSCHECK

- Symington: “This benchmark explores the ability of the software package to accurately predict the first five modes of vibration of thin square plate constrained to act as a cantilever. The plate measures 10 x 10 x 0.05m. An analytical reference solution to this problem is provided in the NAFEMS Publication [“Selected Benchmarks of Natural Frequency Analysis”](#)”.
- StressCheck solution:
 - 254 geometrically mapped tetrahedra, p=4 to 6, <20 seconds run time
 - Converged natural frequency modes and estimated errors are below:

Run	DOF	Natural Freque...	% Error	Run	DOF	Natural Freque...	% Error
1	10470	4.187830e-01	0.20	1	10470	3.341952e+00	2.09
2	19515	4.179751e-01	0.01	2	19515	3.275812e+00	0.07
3	32655	4.179502e-01	0.00	3	32655	3.273620e+00	0.00
1	10470	1.030907e+00	0.71	1	10470	3.793778e+00	1.86
2	19515	1.024018e+00	0.04	2	19515	3.727300e+00	0.07
3	32655	1.023656e+00	0.00	3	32655	3.724749e+00	0.01
1	10470	2.591137e+00	1.12				
2	19515	2.562989e+00	0.02				
3	32655	2.562435e+00	0.00				

Target natural frequencies

Frequencies (Hz)					
	Mode 1	Mode 2	Mode 3	Mode 4	Mode 5
Reference Solution	0.421	1.029	2.582	3.306	3.753

Comment
There is a possibility of poor selection of the master degrees of freedom, because of the identical mass/stiffness terms at the inner nodes.

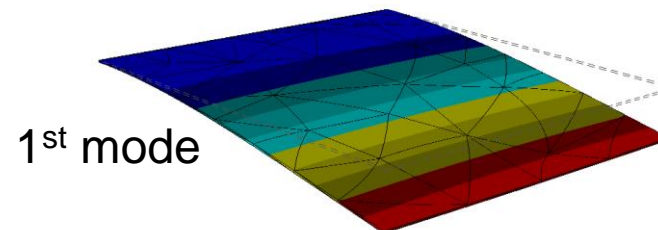


Image courtesy NAFEMS

Benchmark 7: Cantilever Under End Load



STRESSCHECK

- Symington: “On first inspection, the simple cantilever beam bending problem described in this benchmark appears trivial. The reason for including a pure bending problem in this study is because it can often highlight deficiencies in both element formulation and the refinement of the automatically generated mesh. The cantilever in question has a square cross section and is loaded with a distributed force acting on the end face. The target solution can be obtained from an engineering handbook or using general purpose FEA code. Both the peak deflection and bending stress are defined as the targets of this benchmark.”
- StressCheck solution:
 - 125 geometrically mapped tetrahedra, p=2 to 8, <10 seconds run time
 - Max bending stress converges to **226.5 MPa (with 0.14% estimated error)**
 - Max deflection converges to **24.71 mm (with 0.00% estimated error)**

End restraints, reference no.	Boundary values
1a. Left end free, right end fixed (cantilever)	$R_A = 0 \quad M_A = 0 \quad \theta_A = \frac{W(l-a)^2}{2EI}$ $y_A = \frac{-W}{6EI}(2l^3 - 3l^2a + a^3)$ $R_B = W \quad M_B = -W(l-a)$ $\theta_B = 0 \quad y_B = 0$

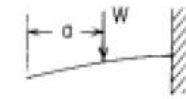
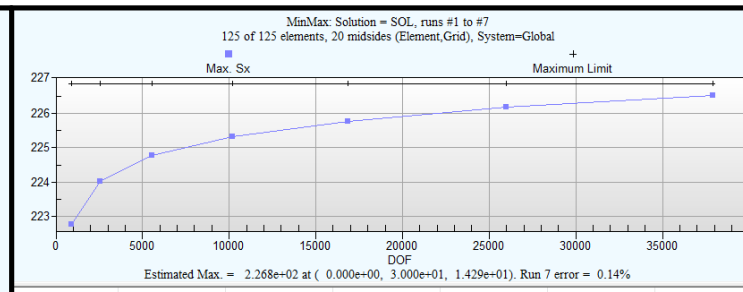
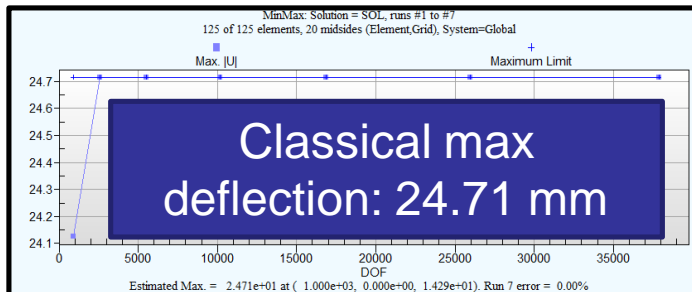


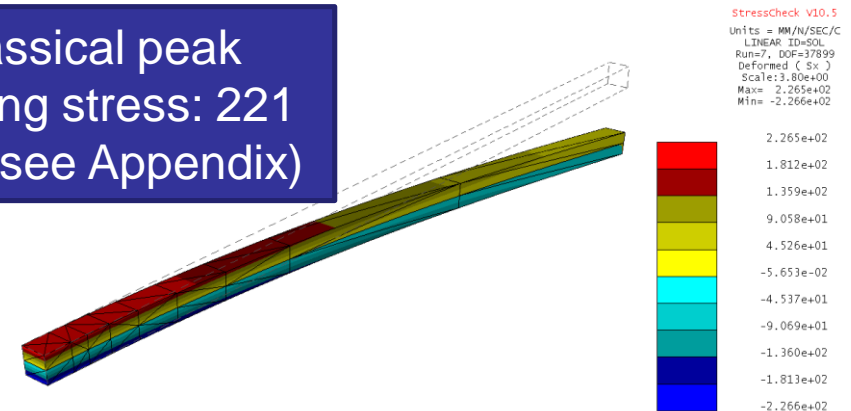
Image courtesy Roark, Table 8.1



Classical peak bending stress: 221 MPa (see Appendix)

Run	DOF	Max. U	Maximum Limit	X (Global)	Y (Global)	Z (Global)
1	918	2.413e+01	2.471e+01	1.000e+03	0.000e+00	2.000e+01
2	2579	2.471e+01	2.471e+01	1.000e+03	3.000e+01	1.714e+01
3	5541	2.471e+01	2.471e+01	1.000e+03	0.000e+00	1.571e+01
4	10179	2.471e+01	2.471e+01	1.000e+03	0.000e+00	1.571e+01
5	16868	2.471e+01	2.471e+01	1.000e+03	0.000e+00	1.571e+01
6	25983	2.471e+01	2.471e+01	1.000e+03	0.000e+00	1.429e+01
7	37899	2.471e+01	2.471e+01	1.000e+03	0.000e+00	1.429e+01

Run	DOF	Max. Sx	Maximum Limit	X (Global)	Y (Global)	Z (Global)
1	918	2.228e+02	2.268e+02	0.000e+00	3.000e+01	0.000e+00
2	2579	2.240e+02	2.268e+02	0.000e+00	3.000e+01	1.143e+01
3	5541	2.248e+02	2.268e+02	0.000e+00	3.000e+01	1.286e+01
4	10179	2.253e+02	2.268e+02	0.000e+00	3.000e+01	1.286e+01
5	16868	2.258e+02	2.268e+02	0.000e+00	3.000e+01	1.000e+01
6	25983	2.262e+02	2.268e+02	0.000e+00	3.000e+01	1.714e+01
7	37899	2.265e+02	2.268e+02	0.000e+00	3.000e+01	1.429e+01



Benchmark 8: Cantilever with Realistic Support



STRESSCHECK

- ❑ Symington: “The geometry used in Benchmark 7 was extended and built into a larger structure with the intention of exploring the stress concentration at the point of connection. A 5mm fillet radius is used to smooth the transition between cantilever and supporting structure. Due to the small size of the fillet radius in regard to the height and length of the component’s major dimensions, accurately capturing the stress concentration is not a trivial task.”
- ❑ StressCheck solution:
 - 317 geometrically mapped tetrahedra, p=2 to 8, 1 minute run time
 - Max Seq converges to **356.9 MPa (with 0.58% estimated error)**

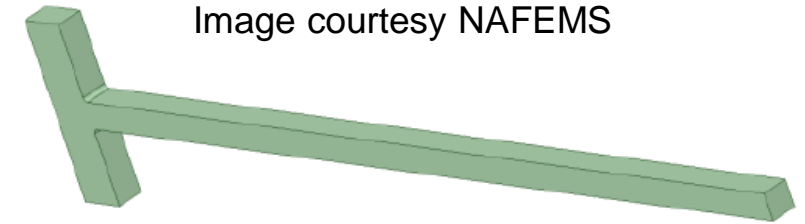
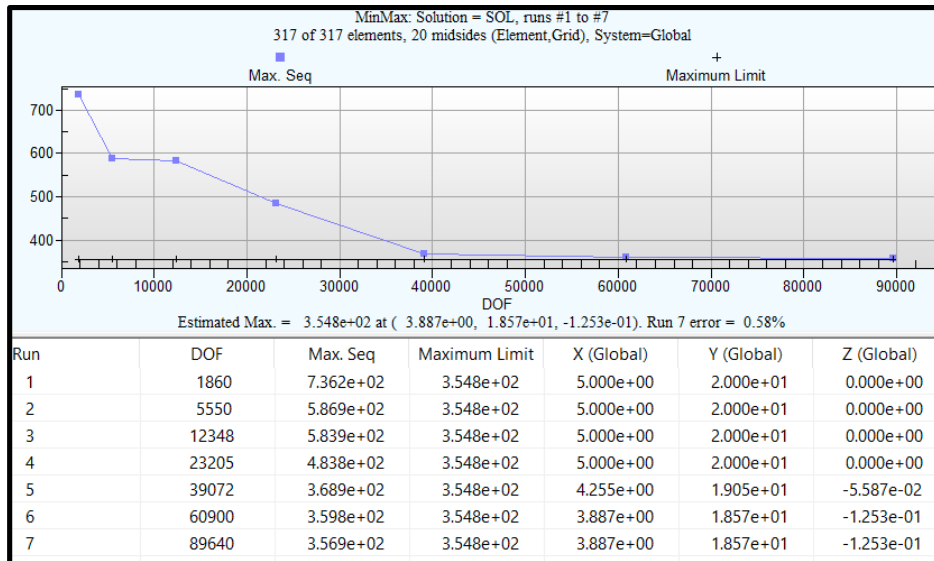
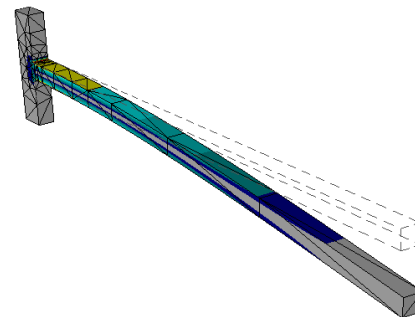


Image courtesy NAFEMS



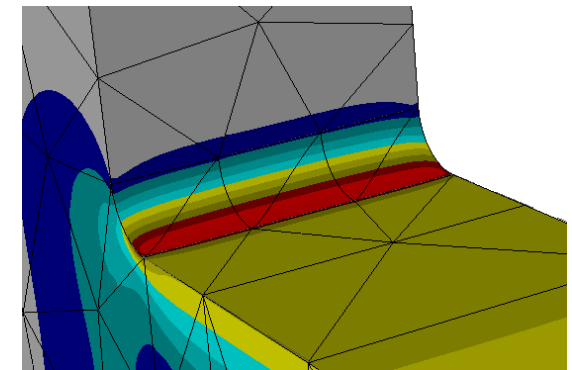
Target max Seq:
356.5 MPa



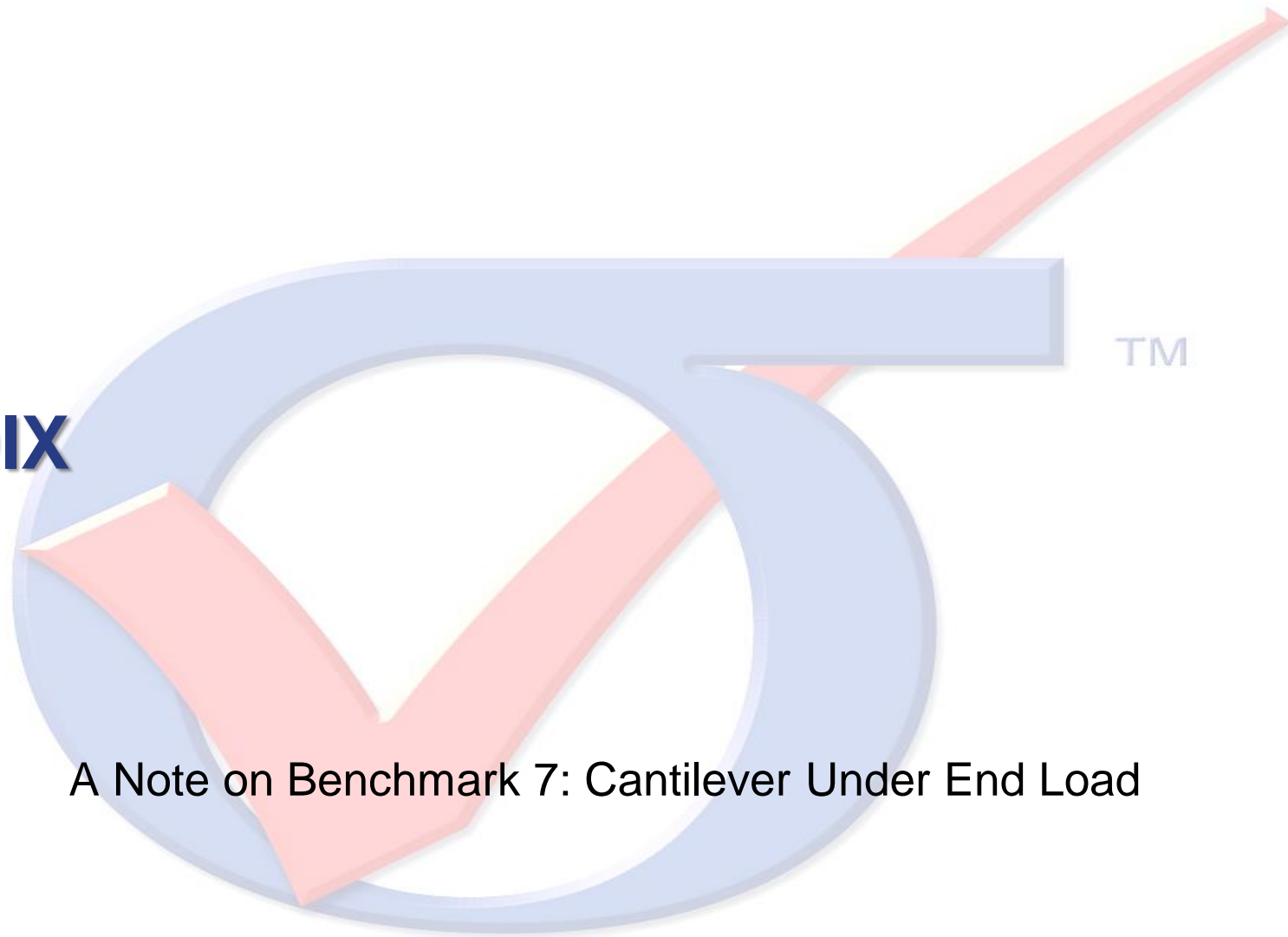
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Max= 3.568e+02
Min= 5.555e-02



3.568e+02
3.211e+02
2.854e+02
2.498e+02
2.141e+02
1.784e+02
1.428e+02
1.071e+02
7.140e+01
3.573e+01
5.555e-02



APPENDIX



A Note on Benchmark 7: Cantilever Under End Load

Benchmark 7 Notes



STRESSCHECK

- ❑ In the classical beam model (quoted from Roark) it is assumed that plane sections remain plane.
 - This is not a valid assumption for the constrained end when solving a problem of elasticity.
- ❑ The StressCheck 3D model assumes self-equilibrated shear loading at the ends, with symmetry at one end (to restrict two rotations) and rigid body constraints on the lower edge (to prevent in plane translations and one rotation).
 - Therefore, the 3D model does not assume a “fixed” end condition as fixing the end would cause stress singularities (stresses at the fixed end would never converge).
 - The 3D model still targets the classical end displacement, and well represents the bending stresses.